Analytical estimation of tooth strength, restored by direct or indirect restorations

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ABSTRACT

This work presents a stress-strain state analytical estimation of restored tooth hard tissues with consideration of their contact interactions with a monolithic filling or a dental onlay. The tooth with a defect is simulated as a continuous isotropic cylindrical body with a non-through hole, and the filling or onlay is simulated as an elastic deformable cylinder. The contact between the tooth and the fillings (onlay) is simulated as the ideal contact. The final strength estimation of the restored tooth is carried out according to the energy criterion. The authors suggest a linear index of tooth damage for practical application of the obtained results. This index can be easily obtained by direct measurement. The differential approach is developed to select a restoring method of the tooth with the defect to provide a reliable restoration of the anatomical crown of the tooth.

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1. Introduction

Thin-walled structures are widespread distributed in nature and widely used in engineering because these structures successfully combine low material consumption and required strength. However, the thin-walled structure like any perfect product does not have a very high safety margin, and therefore is quite sensitive to the violation of nominal load modes and to various kinds of imperfections. Stress concentrators or other defects develop during operation and jeopardize these structures, thus they can significantly reduce structure’s life (Gross & Seelig, 2018). Frequent fractures of anatomical co-root of a tooth with a defect under full load are the clear example of this phenomenon (Jin et al., 2016; Listl et al., 2015). Also, residual stresses and contact loads appearing after restoration can be dangerous when the thin-walled structure contacts with an elastic body (Shatskyi et al., 2018; Velichkovich et al., 2018).

Today, the qualitative restoration of the destroyed anatomical crown of the tooth remains an urgent problem of modern biomechanics and dentistry (Listl et al., 2015). A correct selection of restorative materials and a crown restoring method are the keys to a successful solution to this problem. First, this approach should provide the durability of all elements of the composition (stored tooth and restoration material) and their adhesion. Tooth functional properties significantly depend on the chewing stresses of
hard tissues of the tooth. Therefore, careful calculation of the stress-strain state indicators and the restored
tooth crown strength assessment are necessary in order to choose an adequate restoration method. The
inadequate study of this issue and the inconvenience for the practical application of already existing
results have become the motivation for this research works.

The technology of tooth restoration includes direct and indirect methods of restoration, in particular
filling, pin constructions, dental onlays and crowns. The crowns are obtained by the centrifugal casting
method (Ropyak et al., 2016). The first attempts to analyze the destruction causes of chewing tooth
restored crowns as a biomechanical problem were initiated in the works of Milikevich et al., (1981) and
subsequently developed in the works of Mamoun & Napoletano, (2015); Srirekha & Bashetty, (2013);

The method, proposed in Milikevich et al., (1981), also determines the tooth occlusal surface
destruction index (TOSDI) and recommends using one of the four main types of filling (seal, dental
onlay, artificial crown, pin construction) for tooth crown clinical restoration. The type of used filling
depends on the diagnostic scale (0 – 0.55 – 0.6 – 0.8 – 1) of the TOSDI possible values. The TOSDI
index is defined as the ratio of the 2nd class (by G. V. Black) carious cavity area and the tooth occlusive
surface area. Practical recommendations for a tooth restoration method usage are based on the developed
a mechanic-mathematical model of the tooth with the defect and relation between the tooth crown stresses
and the TOSDI index. However, this index does not take into account destruction depth of the hard tooth
tissues. Therefore, an improved index was proposed in Mamoun & Napoletano (2015), which consider,
additionally, tooth crown fracture depth index (TCFDI) and gives recommendations for tooth restoration
method depending on a two-dimensional diagnostic scale. The works published by Srirekha & Bashetty,
(2013); Nishta et al., (2014) propose a finite-element model derived from the graph approach or three-
dimensional modeling for the stress-strain state of the “tooth-filling” system.

Contact problems for elastic systems constitute the essential part of the mechanics of deformable
solids. Statements and methods for contact problem solving by continuum models of a continuous
medium are presented in the works of different researchers such as (Goryacheva & Martynyak, 2014;
Kravchuk & Neititanaanaki, 2007; Dolgov et al., 1996). Pryhorovska & Chaplinskyy, (2014); Pryhorovska,
(2017); Larson et al. (2013) studied the problems of contact interaction of the tooth and
solids with loading. These works shows the general patterns of contact interaction and describe the state
of the problem. However, these approaches cannot fully meet the needs of engineering practice because
of their mathematical complexity. The works, which describes the numerical methods for contact
problem solving and experimental approaches (for example, (Velichkovich & Dalyak, 2015;
Velichkovich et al., 2011; Vlasiy et al., 2017)) deserve attention too. Attention is paid to the simulation
of strength, stiffness and stability of shell and planar structures, especially of their complex forms
(Panevnik & Velichkovich, 2017) and for cases of stress concentrators (Kolesov et al., 1992) or
inclusions. In particular, the effects of stress concentrations in nonhomogeneous solids near cavities and
inclusions were studied in (Kolesov et al., 1993; Shats’kyi, 2015), and a model was proposed for
assessing the efficiency of rectifying crack-like defects with non-contrast material in (Striletskyy &
Rovinsky, 2017). The physical (Saakiyan et al., 1987), electrochemical (Saakiyan et al., 1989; Dolgov,
2016) and analytical (Shatskii, 1989) methods are used to study the behavior of materials, including
layered structures with loading. However, models and methods of the theory of rods, shells and plates
are more suitable for analytic studies of body contact interactions. According to these approaches,
the main difficulties in contact tasks arise at the stage of research object simulations and the adequate choice
of theory. But the contact tasks, in fact, for thin-walled elements have their own specificity associated
with the emergence of irregular contact tensions, additional zones of detachment, etc. The analytical
methods for solving such problems for plates and shells with defect-kits taking into account the effect of
closing cracks under multiparameter loading were first proposed in the papers (Shatskii, 1989; Shats’kyi
& Makoviichuk, 2005).
In our opinion, the analytical mechanical and mathematical models of the system "tooth-filling", studied in the previous papers, are somewhat simplified and do not consider a number of important factors. First of all, we mean neglecting of the numerical calculations of filling compressibility and not taking into account the effects of Poisson for the tooth tissues, which distorts the parameters of the stress-strain state and does not allow to estimate the possibility of loading influence of the system "tooth-filling" by stretching stresses. Such simplifications can significantly affect the proper choice of the tooth crown restoration method. Therefore, our work presents the results of research aimed at improving the methods of analytical evaluation of the stress-strain state of the system "tooth-filling" or "tooth-onlay". The authors developed the model, based on the idea the model is not universal, but directed to solving a specific problem; however it will make possible to receive analytical results, suitable for convenient use in dentistry. The model is, first, valuable in terms of obtaining a qualitative picture of the phenomena and studied processes. The model may be enriched by the methods (Shatskyi et al., 2016; Ropyak et al., 2017; Shatskii & Perepichka, 2013; Shats'kyi et al., 2016; Velichkovich, 2005) and it will make possible taking into account the layering of the tooth tissues and consider the research with local, arbitrarily oriented and dynamic loads, as well as the fatigue damage. The final quantitative data will be refined by means of point (single) virtual experiments.

The purpose of the study was to improve the analytical evaluation method of tooth crown restoration for the “tooth – filling” or “tooth–onlay” stress-strain state and to develop recommendations for destroyed crown recovery.

The purpose is detailed in the following tasks:
- to develop a mechanical - mathematical model of the “tooth – filling” contact as a system of equations, describing the static equilibrium of the system, physical relations of the tooth and filling solid tissues with external compressive loading, boundary conditions and contact conditions;
- to study the stress-strain state of the system and calculate all its indicators (in particular axial, contact and circumferential tensions);
- to develop and draw relations between the obtained results and the Tooth Linear Damage Index (TLDI), elastic-mechanical properties of the tooth and feeling materials used for the tooth crown restoration;
- to analyse the relations, in particular to evaluate theory, that takes into account maximum equivalent Guber-Mises stress magnitude;
- to develop practical recommendations on tooth restoration rational method choice.

2. Materials and methods of tooth tissue and monolithic filling or dental onlay strength study

The objective of the study is the “tooth – filling (onlay)" stress-strain state definition with consideration contact interaction of the tooth hard tissues with the filling (onlay). The problem of contact interaction of a cylindrical shell (a tooth crown model) and elastic filler (a onlay model) with hard action of a hard stamp (a chewing load model) was formulated and solved. The developed models of the shell and the filler made possible to formulate the boundary problem in a form, suitable for analytical solving. The method of stress-strain state determination and system strength evaluation was developed. Obtained result reliability is confirmed by validity of the geometric-linear problem formulation, the rigorous implementation of mathematical methods tested in the literature for analytical studies, the individual partial (marginal) case result convergence with literature known results.
2.1 Materials, used for experimental study

The study was conducted for a piecewise-homogeneous elastic composition with hard stamp loading, which simulates chewing of the tooth, restored by direct or indirect restorations. The cross-section of the model is a dual-binding region with elasticity and strength ranging from layer to layer, i.e. piecewise-constant functions related the radial coordinate. The stress-strain expressions for all material layers were developed based on the same static and kinematic hypotheses of the elasticity linear theory.

The following statements were assumed:
- material properties of the homogeneous regions are determined by the Young modules, Poisson's coefficients, and the boundaries of yield or strength (Chung & Park 2016).
- properties of tooth hard tissues are considered constant, and the properties of the restoration material are considered discrete-variable. The following materials were used to simulate tooth restoration:
  - a common light hardening filling material (a Gradia Direct type composite) with options: modulus of elasticity \( E=1.2 \) GPa, Poisson coefficient \( \mu=0.43 \).
  - a ceramic material for inserts (type Vitablocs) with the following options: modulus of elasticity \( E=6.0 \) GPa; Poisson coefficient \( \mu=0.4 \).

The averaged elastic constants for the hard tissues of the tooth were: \( E_0=95.0 \) GPa, \( \mu_0=0.32 \).

2.2 Mechanical-mathematical model of the system “tooth-filling” or “tooth-onlay”

Due to complexity of the masticatory tooth occlusive surface and its layered structure, the authors assumed some statements. The restored tooth is simulated as a cylindrical body 1 with a non-through hole (Fig. 1), smoothly stubbed in a rigid obstacle. The model options are:
- vertical wall thickness \( h \);
- outer diameter \( D \);
- hole depth is the same order value with the outer diameter \( D \);

The hole cavity is densely filled with an elastic deformable cylinder 2 (radii \( R \) and height \( a \)). Tooth tissues are considered as a continuous isotropic medium with averaged elasticity constants. A flat smooth area represents the chewing surface. Chewing effort \( Q \) is applied to the tooth surface by the hard stamp with average pressure \( p = Q/(F + F_0) \), where \( F \) – filling (or onlay) cross-sectional area, \( F_0 \) – cross-sectional area of the tooth with a defect. The edge effect in the cylindrical body bottom is neglected at the initial research stage. The stress-strain state of the system is described in the cylindrical coordinates.

Fig. 1. Schematic diagram of the restored tooth: 1 – hard tooth tissues; 2 – filling (onlay)
The elastic equilibrium of the cylindrical body (the model of the tooth with the defect) is described by the momentless shell theory. The tooth static equilibrium is described the relations:

\[ h \frac{d\sigma_{z0}}{dz} - \tau = 0, \]  
\[ h\sigma_{\beta0} = -R\sigma, \]  

(1)  
(2)

The reaction of the loaded tooth hard tissue is described by the physical relationships (generalized Hooke's law):

\[ \varepsilon_{z0} = \frac{1}{E_0} \left( \sigma_{z0} - \mu_0\sigma_{\beta0} \right), \]  
\[ \varepsilon_{\beta0} = \frac{1}{E_0} \left( \sigma_{\beta0} - \mu_0\sigma_{z0} \right) \]  

(3)  
(4)

and the Cauchy relations: \( \varepsilon_{z0} = \frac{du_0}{dz} \), \( \varepsilon_{\beta0} = \frac{w_0}{R} \), where \( \sigma_{z0} \), \( \sigma_{\beta0} \) – axial and circular tooth strains, \( \varepsilon_{z0} \), \( \varepsilon_{\beta0} \) – axial and circular tooth deformations, \( \sigma \), \( \tau \) – normal and tangential contact stresses, \( u_0 \), \( w_0 \) – axial and radial displacements, \( E_0 \), \( \mu_0 \) – average Young's modulus and Poisson's coefficient of tooth hard tissues.

As the initial model relations for the internal cylinder (filling or onlay), we use the following equations of equilibrium (Popadyuk, 2016):

\[ \frac{d\sigma_z}{dz} + \frac{2\tau}{R} = 0, \]  
\[ \sigma_r = \sigma_\beta = \sigma_z. \]  

(5)  
(6)

Hooke's law

\[ \varepsilon_z = \frac{1}{E} \left( \sigma_z - \mu(\sigma_r + \sigma_\beta) \right), \]  
\[ \varepsilon_\beta = \frac{1}{E} \left( \sigma_\beta - \mu(\sigma_r + \sigma_z) \right), \]  

(7)  
(8)

and the Cauchy relations: \( \varepsilon_z = \frac{du}{dz} \), \( \varepsilon_\beta = \frac{w}{R} \), where \( \sigma_r \), \( \sigma_\beta \), \( \sigma_z \) – radial, circumferential and axial stresses in the filling (or onlay); \( \varepsilon_z \), \( \varepsilon_\beta \) – radial, circumferential and axial deformations of the filling; \( u \), \( w \) – radial and axial displacement of the filling, \( E \), \( \mu \) – the elastic modulus and Poisson's ratio of the material of the filling (onlay). The ideal contact relations describe the tooth and filling (onlay) contact conditions: \( u_0 = u \), \( w_0 = w \). The axial and radial displacement equality conditions \( u_0 = u \) and \( w_0 = w \) are replaced by the compatibility conditions of the tooth and filling (onlay) deformations. The boundary conditions of the tooth free face end are:

\[ \sigma_z(0)F + \sigma_{z0}(0)F_0 + Q = 0. \]  

(9)
2.3 “Tooth-filling” and “tooth-onlay” stress-strain state indicator definition

The aforementioned stresses $\sigma_{z0}$, $\sigma_{\rho0}$, $\sigma_r$, $\sigma_\rho$, $\sigma_z$, are the main indicators of the “tooth-filling” and “tooth-onlay” stress-strain state. In order to find them, the relations (3–7) were transformed:

$$
\begin{align*}
&h\sigma_{\rho0} = -R\sigma, \\
&\sigma_r = \sigma, \sigma_\rho = \sigma, \tau = 0, \\
&\frac{1}{E}\sigma_z - \frac{1}{E_0}\sigma_{z0} - \left(\frac{2\mu}{E} + \frac{\mu_0 R}{E_0 h}\right)\sigma = 0, \\
&-\frac{\mu}{E}\sigma_z + \frac{\mu_0}{E_0}\sigma_{z0} + \left(\frac{1}{E_0 h} + \frac{1-\mu}{E}\right)\sigma = 0.
\end{align*}
$$

(10)

All stress state components were defined by the system (10) solving with the boundary condition (9). The main analytical results are presented below.

Axial stresses in the filling are:

$$
\sigma_z = -\frac{Q}{F_0} \cdot \zeta,
$$

(11)

where

$$
\zeta = \frac{1}{E_0} \left(\lambda - \mu_0\right) + \frac{1}{E_0 F_0} \left(\lambda - \mu_0\right), \quad \lambda = \frac{2E F_0 + (1-\mu) E_0}{2 \left(\mu E_0 + \mu_0 E \frac{F}{F_0}\right)}.
$$

Axial stresses in the tooth hard tissues are:

$$
\sigma_{z0} = -\frac{Q}{F_0} \cdot \left(1 - \frac{F}{F_0} \zeta\right).
$$

(12)

The normal stresses on contact surfaces of fillings and tooth (contact pressure) are:

$$
\sigma = \frac{Q}{2E_0 + (1-\mu)\frac{F_0}{E}} \cdot \left(\mu_0 - \zeta \left(\frac{\mu}{E_0} \frac{F}{F_0}\right)\right).
$$

(13)

The circumferential stresses in the tooth hard tissues are:

$$
\sigma_{\rho0} = -2\sigma \frac{F}{F_0}.
$$

(14)
2.4 “Tooth-filling” and “tooth-onlay” stress-strain state indicator calculation results and their analysis

The obtained results made possible to specify the following cases. The Tooth Linear Damage Index (TLDI) \( k = h / D \) was used to describe the obtained numerical data. This index is convenient for practical usage, because of its easiness for calculations (Fig. 1). Fig. 2 depicts tooth hard tissue axial stresses related the TLDI. The more rigid material takes more share of external axial loading. Limit transitions by the expression (12) show the following:

- for the tooth with the defect (without restoration) \( E \to 0 \) and \( \mu \to 0 \), was obtained \( \sigma_{z0} = -Q / F_0 \);
- for the healthy tooth \( E = E_0 \) \( \tau_0 = \mu_0 \), was obtained \( \sigma_{z0} / p = -1 \);
- if \( k = h / D = 0.5 \), then \( \sigma_{z0} / p = -1 \), which also corresponds to the healthy tooth.

Fig. 3 depicts the contact stresses of the filling (onlay) external surface and the tooth internal surface related the TLDI index. It worth mentioning, contact stresses were negative (compressing) for both a photopolymer filling and a ceramic onlay. An additional analysis of the formula (13) shows the following:

- if the filling with \( \mu < \mu_0 \) was used for restoration, the tensile stresses will change their sign - they will become stretching, and it can lead to a deterioration of adhesion or even the separation (removal) of the filling from the tooth;
- for the healthy tooth \( E = E_0 \) \( \tau_0 = \mu_0 \), it was obtained \( \sigma = 0 \);
- contact stresses were, averagely, two orders lower, then the axial ones; so they can be neglected in strength evaluation (one of the main stresses of the tooth material is taken to be zero);
- the simplified tooth model without Poisson's effect consideration \( \mu_0 = 0 \) gives significantly overpriced results regarding to the contact stresses.

Fig. 4 depicts the circumferential stresses of the restored crown tooth tissues related the TLDI index. These tensions were positive, that is, stretching. In both cases, the following was observed: the smaller thickness of the tooth crown wall \( h \), the higher the ring stresses; the intensity of circumferential tension increasing related to the TLDI index is accompanied by increasing of the Poisson coefficient of the restoration material. The stressed state of the loaded hard tissues of the restored tooth is complex. Therefore, one of the theories of strength was used to assess the strength, and the actual estimate is to be